Does the Violation of Bell's Inequality Refute All Local Realisms

By

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While Bell's theorem and its proof via the violation of Bell's inequality are considered irrefutable proof of the non-locality of nature at the microscopic scale it is, to be precise, a proof that nature doesn't conform to a particular definition of local realism as defined in the EPR paper. But does the proof of Bell's theorem refute other definitions of local realism which may better describe reality?

Bell's theorem states:

No theory of local hidden variables can reproduce all of the predictions of quantum mechanics.

In axiomatic terms, Bell's theorem is simply a special case of the more general theorem:

It is not possible to derive all predictions of a theory from a second theory when the theories considered are based on mutually exclusive axiom sets.

When applied to quantum mechanics and a hidden variable theory, all that is required to prove Bell's theorem is to find two mutually exclusive axioms, one belonging to the axiom set of the hidden variable theory and the other belonging to quantum mechanics. But such a proof would do nothing to answer the question of completeness of quantum mechanics.

In order to answer that question, the usual proof of Bell's theorem proceeds first with a generalization of hidden variables theories and derives from it a prediction of the probabilities of coincidences of measurements; what we call Bell's inequality. Since the predictions of Bell's inequality are distinct from that which is derived from quantum mechanics, the question is to be settled by experiment.

All experiments violate Bell's inequality and support quantum mechanics prediction therefore are understood to provide the definitive answer to the question as to whether reality is local or non-local (though they do not provide a definitive answer as to the completeness of quantum mechanics).

While the proof of Bell's theorem is the inevitable consequence of the generalization of hidden variables theories from which Bell's inequality is derived, the generalization itself must be questioned.

Hidden Variables States

The key implicit assumption from which is derived Bell's inequality is that possible states of a particle as defined by hidden variables is unaffected by measurement. The definition of a state is given as one that will determine whether a property will or will not be detected by one or more detectors. For a detector D_1, there are two possible states; pass or fail. For three tests, D_1, D_2 and D_3, we thus have eight possible states (see table 1). As one can see, the number of possible states is dependent on the number of tests that one can perform[1].

The possible measurements of particles having the eight possible states are tabulated and from them is derived Bell's inequality which constrains the predictions of derived from any hidden variable theories. A simple example of this is given below.

In table 1, we have the eight possible states as their ability to pass or fail tests D_1, D_2 and D_3. Table 2 shows labels "1" when two tests performed on particles having the same state both pass or both fail the test.

Table 1			
state	D_1	D_2	D_3
1	+	+	+
2	+	+	-
3	+	-	+
4	+	-	-
5	-	+	+
6	-	+	-
7	-	-	+
8	-	-	-

Table 2				
state	D_1D_2	D_2D_3	D_1D_3	prediction
1	1	1	1	1
2	1	0	0	1/3
3	0	1	0	1/3
4	0	0	1	1/3
5	0	0	1	1/3
6	0	1	0	1/3
7	1	0	0	1/3
8	1	1	1	1

Ignoring the cases where all measurements are the same, which doesn't provide any information allowing us to distinguish the properties of the particles are left with measurements 2 to 7 of the correlation table. The probability of have two particles either passing or both failing two distinct tests is 1/3 or .333. This is in disagreement of quantum mechanics prediction of .250. Experiments confirm that quantum mechanics prediction is correct.

This is assumed to refute hidden variables, but the description of hidden variables ignores an important physical influence: the effect of the detectors themselves on the state of a particle. We will show now that by assuming a local effect of detectors on the state of particles as defined in Bell's paper we can derive predictions that are not only consistent with the

[1] *Note: This dependence of the number of states on the number of tests is a strong theoretical bias since it implicitly contains quantum mechanics' definition of reality.*

measurements of Bell experiments, but also provide a local realistic explanation of the measurements of so-called quantum entanglement experiments.

To avoid any misleading mathematical abstractions and keep to as strict a physical interpretation as possible, we will only consider Stern-Gerlach experiments on electrons and the measurements of the directional components of the property of spin understood as determining whether or not an electron will pass the detectors of the apparatus.

Here, we will work from the assumption that detectors affect the property of the particle it measures locally (which qualifies as a hidden variable), then we will derive predictions in a manner similar to how Bell's inequality is derived from the assumption of invariable states.

Let us assume three types of detectors: D_1, D_2 and D_3 which relative orientations are such that $\angle D_1D_2 = \theta_{12}$, $\angle D_2D_3 = \theta_{23}$ and $\angle D_1D_3 = \theta_{13}$. Let us assume a group of n_0 electrons from a source which spins are not polarized.

When passing through D_1, all of the n_0 electrons are subjected to the detector's magnetic field. Let's assume that each electron interact with the magnetic field in a way that depends on the orientation which in turn determine its resulting change in its trajectory. Let's assume that in addition to a change in trajectory, the electrons also experience changes in the directional components of their spins.

The orientation of the spin of an electron can have any value from 0 to 2π relative to the axis of D_1. If the magnetic field is composed of polarized particles moving in at angles between 0 and π and between π and 2π, they may be absorbed by electrons which orientations have orientations respectively between 0 and π and between π and 2π .

The detector is set up so that electrons which relative spin angle are between 0 and π, will pass through and the electrons with spin between π and 2π will not.

So for a group of n electrons with random spin orientation, we can predict that n_1, the number of electrons that will pass D_1, is given by $n_1 = n_0 \frac{\pi}{2\pi} = \frac{n_0}{2}$ and all n_1 electrons with have directional components of spin between 0 and π. So half of the electrons will pass through D_1 .

Now, if those n_1 electrons are directed to a second detector, D_2, their orientation relative to the axis of D_2 will be from θ_{12} to $\theta_{12} + \pi$ (see figure 1b below).

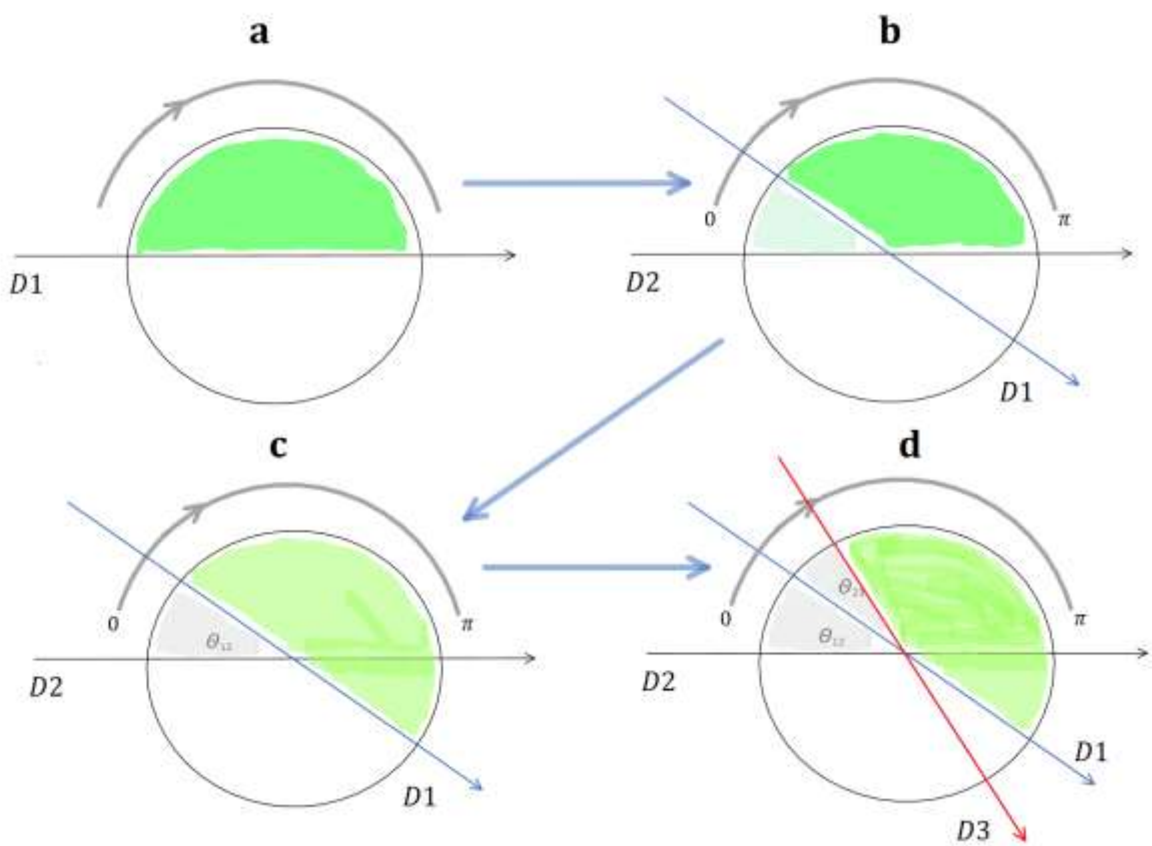

Figure 1

Here we see that n_2 , the number of electrons that will pass D_2 must be $n_2 = n_1 \frac{\pi - \theta_{12}}{2\pi}$.

Now, let's assume that $\theta_{12} < \pi$ so that $n_2 > 0$, as the electrons pass through the magnetic field of D_2, they will be interacting with the particles composing the magnetic field which the angles of motion are between 0 and π degrees relative to the axis of D_2. That is, it will impart the electrons with changes in momentum that are between 0 and π degrees relative to the axis of D_2. The n_2 electrons will have directional components relative to the axis of D_2 that will vary between 0 and π degrees (see figure 1c).

Consequently (figure 1d), when the n_2 electrons enter D_3, the number of electrons that pass this last detector is given by $n_3 = n_2 \frac{\pi - \theta_{23}}{2\pi}$.

In the special case (figure 2) where $\theta_{12} = \pi$, we would find $n_2 = n_1 \frac{\pi - \pi}{2\pi} = 0$. That is, no particles would pass through D_2 if it axis correspond to a rotation of π degrees from the axis of D_1 .

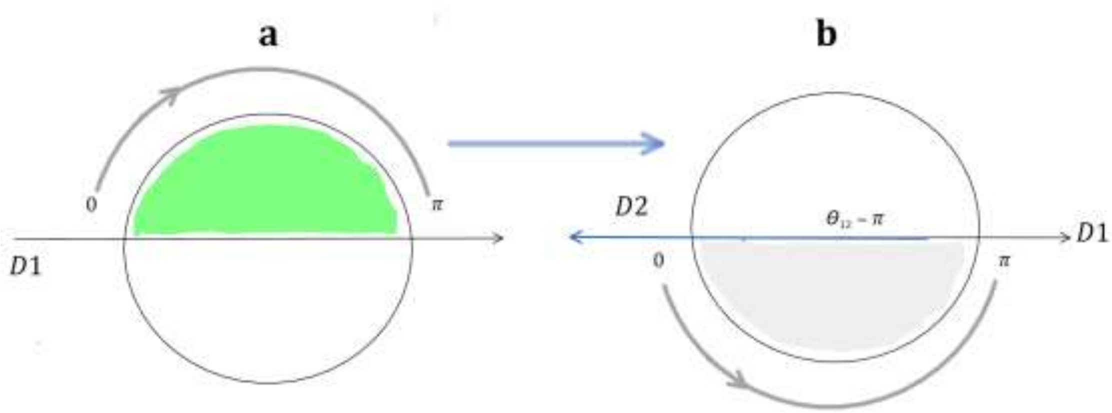

Figure 2

But if between two detectors at π degrees angles we introduce a third detector at angle $\theta_{12} < \pi$, we find that $n_2 = n_1 \frac{\pi - \theta_{23}}{2\pi} > 0$ (figure 3d).

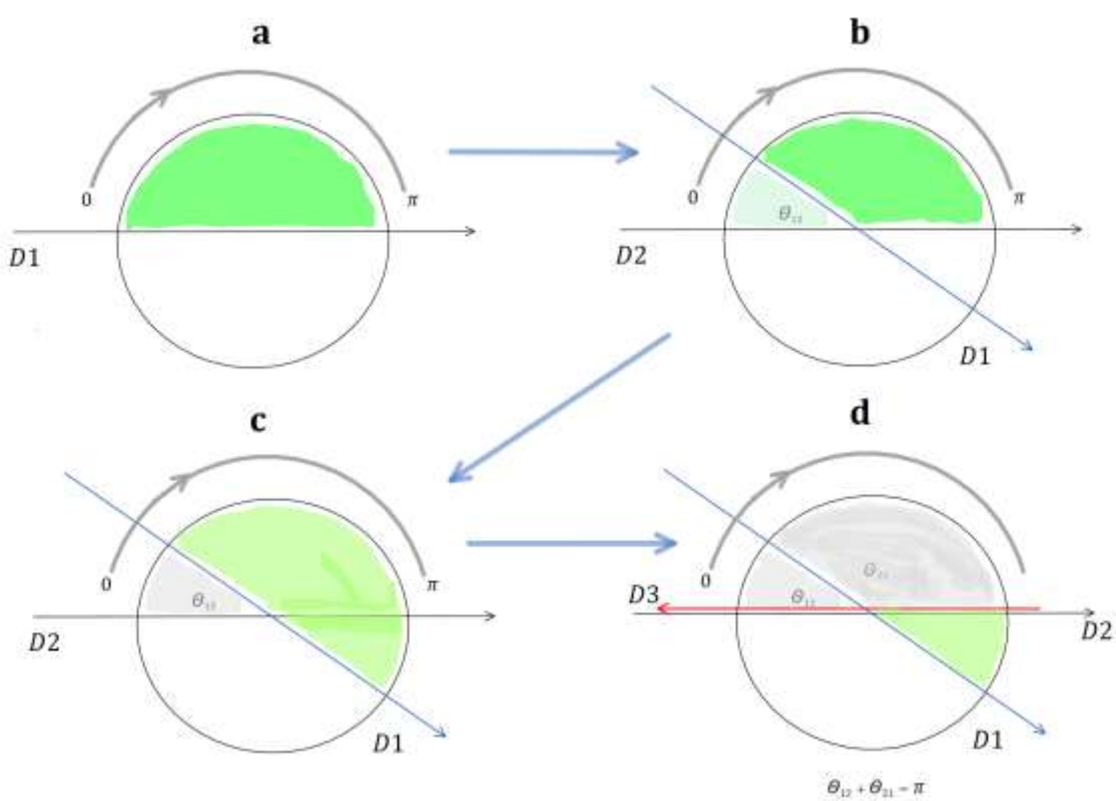

Figure 3

This explains classically why though $n_1 = 0$ when $\theta_{12} = \pi$, we will have $n_2 > 0$ when $\theta_{12} < \pi$ and $\theta_{23} = \pi - \theta_{12}$, that even though $\theta_{12} + \theta_{23} = \pi$. We can thus use the above hidden variables to predict deterministically the number of electrons that will pass through any series or arrangements of detectors.

In order to prove the above, all that is needed is to test the predictions for Stern-Gerlach experiments derived from the general equation $n_x = n_0 \prod_{i=1}^{x} \frac{\pi - \theta_{i-1,i}}{2\pi}$ where n_0 is the initial number of unpolarised electrons from a source and $\theta_{i-1,i}$ is the angle between the detector D_i and the previous detector D_{i-1} in the sequence $D_1 \to D_2 \to D_3 ... \to ... D_X$ for $i > 1$ and where since there is no D_0 for $i = 1$ we have $\theta_{0,1} = 0$.

Let us know examine the proposed generalisation of hidden variable theories given in table 1.

Table 1			
state	D_1	D_2	D_3
1	+	+	+
2	+	+	-
3	+	-	+
4	+	-	-
5	-	+	+
6	-	+	-
7	-	-	+
8	-	-	-

From the simple assumption that the orientation of the spin is affected by the magnetic field of detectors we find that:

The particle in state 1 is a special case where θ_{s_1} the angle of the electron spin relative to D_1 is between 0 and π and $\theta_{12} < \theta_{s_1}$ and $\theta_{23} < \theta_{s_2}$ where θ_{s_2} is the angle relative to $D2$ after interacting with its magnetic field if a single electron passes successively through the detector in the exact (not reversible sequence) $D_1 \to D_2 \to D_3$ or if table 1 represents distinct electrons going through each detector, which we will represent by $D_1 | D_2 | D_3$, then $\theta_{12} + \theta_{23} < \theta_s < \pi$.

State 2 is a special case where θ_{s_1} the angle of the electron spin relative to D_1 is between 0 and π and $\theta_{12} < \theta_{s_1}$ if $D_1 \to D_2 \to D_3$. If $D_1 | D_2 | D_3$ then $\theta_{12} < \theta_s < \pi$ and $\theta_s > \theta_{12}$.

In fact, we can show that all the states which together are assumed to generalize hidden variables represent a small subset of all possible states when possible angles of spin, relative angles of detectors, their sequences and other physical characteristics (such as the density of their magnetic field) are taken into account.

Thus if the assumption regarding the interaction between electrons and detectors is correct (that is: experimentally supported), then the set of states used to derive Bell's inequality is not a generalization of hidden variable theories. Since experiments are consistent with the predictions derived from the above assumption, it follows that Bell's inequality does not constrain hidden variables theories. Consequently, the violation of Bell's inequality does not refute local realism.

If Bell's generalization of hidden variables is incomplete then the inequality derived from it loses all meaning. As a consequence, whatever Bell experiments are conducted and whatever their results, they would be irrelevant to the question posed by the EPR paper. Determining whether nature is local or non-local may come down to whether or not some predictions made using our model are distinct from some predictions of quantum mechanics and whether or not such distinct predictions are consistent with experimental data.

The model we introduced here can be used make predictions for the outcomes of any arrangement of detectors. The predictions are deterministic rather that probabilistic. For experiments where each particle is measured once, the predictions are equivalent to quantum mechanical descriptions. That is, $p_{D_x|D_y} = .25$ regardless of the orientation of D_x and D_y. Hence such experiments cannot distinguish between our model and quantum mechanics. But for experiments using arrangements where particles are tested more than once, our predictions differ significantly from those of quantum mechanics. For multiple measurement experiments, the probability of equal outcome of two series of measurements is given by:

$$p_{\left(D_1 \to \ldots \to D_x | D_1 \to \ldots \to D_y\right)} = \prod_{i=1}^{x} \frac{\pi - \theta_{i-1,i}}{2\pi} \prod_{j=1}^{y} \frac{\pi - \theta_{j-1,j}}{2\pi}$$

Thus multiple measurement experiments can answer the EPR question.

Also, that a hidden variables theory can predict with certainty the outcome of so-called quantum entanglement experiments without quantum entanglement puts into question the existence of the quantum entanglement and should at the very least hint that quantum mechanics is, as Einstein, Podolsky and Rosen suggested, incomplete.